To: ACI Foundation, Concrete Research Council

From: Dr. Brock Hedegaard and Dr. Mija Hubler

Date: March 31, 2022

Re: Deliverable Phase 4: Final Reporting

Project: CRC 2020 P0034 – *Calibration of Simplified Creep and Shrinkage Models Developed Using Solidification Theory*

This deliverable satisfies the Deliverable Phase 4: Model Selection due March 31, 2022 for contract CRC 2020 P0034 – *Calibration of Simplified Creep and Shrinkage Models Developed Using Solidification Theory*. Work on this project commenced on September 1, 2020. The original due date of August 31, 2021 was granted a no-cost extension until March 31, 2022. This final deliverable thereby concludes the project requirements.

Phase 4: Final Reporting – Shrinkage and Creep Model

Industry Panel Engagement and Model Form Selection

During the industry panel meeting held on June 30, 2021, we presented four different candidate shrinkage models and discussed the proposed form of the creep model.

Our project goal was to deliver a model that was accurate but relatively simple to use for designers who may not have all the information on detailed mix proportions. The four candidate shrinkage model forms are listed from most complex to simplest:

- Candidate Shrinkage Model 1 – Solidification with Coupled Pore Humidity
- Candidate Shrinkage Model 2 – Coupled Pore Humidity Model with Added Aging Term
- Candidate Shrinkage Model 3 – Coupled Pore Humidity
- Candidate Shrinkage Model 4 – Additive Model

The industry panel believed Candidate Shrinkage Model 3, the Coupled Pore Humidity Model, was a desirable model from a usability perspective. The research team agreed with this assessment, and further acknowledged that Model 3 had some theoretical advantages compared to the simpler additive Model 4. Based on preliminary fitting of each model form, all models had roughly similar predictive capabilities. **Therefore, the research team selected Model 3 for the final shrinkage model.**

Also during this meeting, the research team posed the following question to the industry panel regarding the creep model: Would you prefer that the creep model use a traditional aging elastic modulus (similar to most other creep models) to describe the initial strains, or would you prefer to use a nonaging instantaneous modulus (similar to model B4) to describe initial strains? The primary advantages of the aging elastic modulus are familiarity and ease of conversion to a creep coefficient if desired. The advantage of the instantaneous modulus is that initial strain computations are independent of the loading age, and the model likely fits short-term creep better. Neither formulation was expected to be superior for long-term creep predictions, which are often of most interest for design. The industry panel preferred to keep the traditional aging elastic modulus. **Therefore, the research team selected the creep model form with the aging elastic modulus.**

Required Model Inputs

The proposed model is intended for design office purposes, and therefore relies on model inputs that are either known or may be assumed by the designer. **This memo presents all equations in metric units (MPa, mm, and °C), as that was the unit system of the database used in model development and calibration.** English conversions of these equations will be made available on request.

The following inputs are necessary, with suggested values given if not known:

- Mean 28-day concrete strength f_{cm} (MPa). If design strength f_c' is given, then $\boldsymbol{f_{cm} = f_c' + 8}$ **MPa**.
- Aggregate volume ratio g (unitless). May calculate from mix design, but if unknown, then estimate based on strength: $\boldsymbol{g = 0.707 - f_{cm}/1250}$.
- Cement type: normal hardening (Type I) or rapid hardening (Type III). Other cement types may be assumed as the more similar of these two, or may be fit using available shrinkage and creep data for that specific application.
- Curing temperature T_{cur} (°C). If unknown, assume T_{cur} = 20°C.
- Average ambient temperature T (°C). If unknown, assume T = 20°C.
- Average ambient relative humidity h_0 (unitless). Provided as a decimal between 0 and 1.
- Volume-to-surface ratio V/S (mm).

Temperature Corrected Time Variables

This time-dependent model adjusts the time variables for different ambient temperature and curing temperature conditions. The two temperature correction factors are given as:

$$R_0 = \exp\left[U\left(\frac{1}{293} - \frac{1}{T_{cur} + 273}\right)\right] \tag{1}$$

$$R_T = \exp\left[U\left(\frac{1}{293} - \frac{1}{T + 273}\right)\right] \tag{2}$$

where U = 2500 Kelvin is a activation energy constant, T_{cur} is the curing temperature in degrees Celsius, and T is the ambient temperature after curing in degrees Celsius. If temperatures are unknown, or if temperatures are typical room temperature conditions (i.e., $T_{cur} = T$ = 20°C), then both $R_0 = R_T = 1$.

The adjusted time variables are as follows:

Description	Base Variable	Adjusted Time
Time of curing	t_c	$t_{cT} = R_0 t_c$
Time of loading	t_0	$t_{0T} = t_{cT} + R_T(t_0 - t_c)$
Current time	t	$t_T = t_{cT} + R_T(t - t_c)$

The above expressions assume that both t and t_0 are greater than t_c, which is typically the case for design applications. If $t_0 < t_c$, then $t_{0T} = R_0 t_0$, and if $t < t_c$, then $t_T = R_0 t$. If both $R_0 = R_T = 1$, then the adjusted time variable are equal to the corresponding base time variables.

The following shrinkage and creep expressions will be presented in terms of the adjusted time variables.

Final Shrinkage Model

The pore relative humidity due to self-desiccation alone is assumed to follow:

$$\Delta H_{au}\left(t_T\right) = A\ln\left(\frac{t_T - t_v}{B} + 1\right) \tag{3}$$

where t_v = 0.25 days is the duration of the water vapor saturation stage required to consume excess water and begin self-desiccation (Ding et al. 2019), and A and B are fitting parameters. For $t_T < t_v$, pore relative humidity is assumed to be saturated, that is $\Delta H_{au} = 0$.

Comparing this equation to data from Jiang et al. (2006), parameters A and B can be written as function of the mean 28-day concrete strength f_{cm} (units of MPa):

$$A = 0.015 + \frac{f_{cm}}{6000} \tag{4}$$

$$B = 10^{\left(\frac{25}{\sqrt{f_{cm}}} - 4\right)} \tag{5}$$

The pore relative humidity due to drying alone is assumed to follow:

$$\Delta H_{dry}\left(t_T\right) = 0.5\left(1 - h_0^{\;2}\right)\tanh\left(\sqrt{\frac{t_T - t_{cT}}{\tau_{dry}}}\right) \tag{6}$$

For $t_T < t_{cT}$, no drying has yet occurred, so $\Delta H_{dry} = 0$. The shrinkage half-time τ_{dry} (units of days) is proportional to the volume-to-surface ratio V/S (units of mm) squared:

$$\tau_{dry} = 0.08\left(k_s \frac{V}{S}\right)^2 \tag{7}$$

The shape factor k_s depends on the shape of the concrete member, and is equal to:

$$k_s = \begin{cases} 1.00 & \text{infinite slab} \\ 1.18 & \text{infinite cylinder} \\ 1.22 & \text{infinite square prism} \\ 1.28 & \text{sphere} \\ 1.40 & \text{cube} \end{cases} \tag{8}$$

Most solid rectangular beams can be adequately modeled using k_s approximately equal to 1.2, though box girder walls may be more closely approximated as slabs with k_s nearer to 1.0.

The coupled change in pore relative humidity due to both self-desiccation and drying is:

$$\Delta H = \Delta H_{au} + \Delta H_{dry} - \Delta H_{au}\Delta H_{dry} \tag{9}$$

The coupled change in pore relative humidity drives shrinkage (both autogenous and drying):

$$\varepsilon_{sh} = -p\Delta H \tag{10}$$

where the negative sign implies a reduction in volume, and p is the shrinkage coefficient equal to:

$$p = \frac{0.075}{\sqrt{f_{cm}}}(1-g)^{1.7} \tag{11}$$

The same expression is used for either autogenous shrinkage or total shrinkage conditions; the only difference is that $\Delta H_{dry} = 0$ for autogenous shrinkage tests. See description of model inputs for estimates of mean compressive strength f_{cm} and aggregate volume fraction g if unknown.

Final Creep Model

For constant applied stress σ applied at time t_0, the total strain is equal to:

$$\varepsilon = \sigma J(t_T, t_{0T}) + \varepsilon_{sh} \tag{12}$$

The compliance function $J(t_T,t_{0T})$, derived via a creep rate per solidification theory, is equal to:

$$J(t_T, t_{0T}) = \frac{1}{E_{ct0}} + R_{LL}\left[A_c R_T \ln\left(\frac{t_T - t_{0T}}{\beta} + 1\right) + B_c R_T \ln\left(\frac{t_T}{t_{0T}}\right) + p_5\left[\Delta H(t_T) - \Delta H(t_{0T})\right]\right] \tag{13}$$

The elastic modulus at age of loading E_{ct0} is computed based on the concrete strength f_{ct0} at the age of loading t_{0T}:

$$f_{ct0} = f_{cm}\left(\frac{t_{0T}}{a + bt_{0T}}\right) \tag{14}$$

$$E_{ct0} = 4734\sqrt{f_{ct0}} \tag{15}$$

where constants a and b are cement type dependent:

- For Type I cement, $a = 4.00$, $b = 0.85$
- For Type III cement, $a = 2.30$, $b = 0.92$

Adjustment factors R_{LL} and R_T account for nonlinear effects due to high levels of loading and temperatures, respectively. The temperature adjustment factor is given in Eqn. (2), and the nonlinear load level factor is given by:

$$R_{LL} = \begin{cases} 1 & \text{for } \dfrac{\sigma}{f_{ct0}} \le 0.5 \\ \exp\left(\dfrac{\sigma}{f_{ct0}} - 0.5\right) & \text{for } \dfrac{\sigma}{f_{ct0}} > 0.5 \end{cases} \tag{16}$$

where σ is the applied stress.

Creep parameters include viscoelastic compliance A_c, flow compliance B_c, and drying creep compliance p_5 (all with units of MPa^{-1}), and two time parameters K and β:

$$A_c = p_3\left(1+\frac{1}{Kt_{0T}}\right) \quad B_c = p_4 - \frac{p_3}{Kt_{0T}} \tag{17}$$

$$p_3 = \frac{12.5\times10^{-6}}{f_{cm}^{\ 0.7}} \quad p_4 = \frac{30.0\times10^{-6}}{f_{cm}^{\ 0.5}} \quad p_5 = \frac{0.023}{f_{cm}^{\ 0.9}}\left(1-g\right)^{1.7} \tag{18}$$

$$K = 0.25 \text{ days}^{-1} \quad \beta = 0.01 \text{ days} \tag{19}$$

The final p_5-term in Eqn. (13) represents the drying creep, which has the same functional form as the change in pore relative humidity ΔH; see Eqn. (9). Unique among available creep models, this model predicts “drying creep” even for basic creep tests due to self-desiccation; this can capture the reduction in total creep (and basic creep) observed in pre-dried specimens.

The same expressions are used for either basic creep or total creep conditions; the only difference is that $\Delta H_{dry} = 0$ for basic creep tests, though $\Delta H_{au} > 0$.

If a creep coefficient formulation is desired, the creep coefficient ϕ is given by:

$$\phi\left(t_T, t_{0T}\right) = E_{ct0} J\left(t_T, t_{0T}\right) - 1 \tag{20}$$

$$\phi\left(t_T, t_{0T}\right) = E_{ct0} R_{LL}\left[A_c R_T \ln\left(\frac{t_T - t_{0T}}{\beta}+1\right) + B_c R_T \ln\left(\frac{t_T}{t_{0T}}\right) + p_5\left[\Delta H\left(t_T\right) - \Delta H\left(t_{0T}\right)\right]\right] \tag{21}$$

This model satisfies all criteria set forth in the original proposal, namely that it can be expressed as an analytical expression in both integral-type and rate-type formulations, satisfies solidification theory, and is nondivergent (Hedegaard 2020).

Final Swelling Model

The swelling model proposed herein is only applicable for concrete submerged in water. Under such conditions, $\Delta H = 0$; there is self-evidently no drying, and pore relative humidity lost to self-desiccation is also presumed to be replenished. The number of tests in the database under this condition are relatively limited, but available long-term data (Brooks 1984) indicate the swelling curve is a power law (Rasoolinejad et al. 2019):

$$\varepsilon_{sw} = p_{sw}\left(t_T - t_{cT}\right)^{0.2} \tag{22}$$

Insufficient data exist to evaluate how swelling varies based on *V/S* or even concrete strength. A good estimate of the database was achieved by setting the swelling coefficient as a constant: $p_{sw} = 40 \times 10^{-6}$.

Model Comparisons

The proposed model was compared to other design-office, strength-based models: the previous ACI 209 model (ACI 209, 1982), GL2000 (Gardner and Lockman 2001), fib Model Code 2010 (*fib* 2013), and the simplified strength formulation of B4, known as model B4s (RILEM TC-242-MDC 2015). Weighted coefficients of variation were computed for each model. Weighting was performed such that each logarithmic interval of time (0 to 4 days, 4 to 16 days, 16 to 64 days, and so on by powers of 4) was equally weighted, and within each logarithmic interval each test had equal weight. Most data in the database are for tests of duration one year or less. Weighting ensured that the coefficient of variation would not be biased towards fitting only the early age behavior, and no individual test would dominate within each logarithmic interval.

Table 1 summarizes the coefficients of variation for shrinkage computed using the final shrinkage model and each of the historical models. Note that ACI 209 and GL2000 models predict only the total shrinkage, and do not distinguish between autogenous and drying shrinkage; therefore, these entries have N/A for a coefficient of variation. The proposed shrinkage model consistently has lower coefficients of variations than all comparable historical models. The superiority of the proposed model even holds when computing the coefficient of variation for the more limited dataset that conforms to the published limits of applicability of the B4 model (RILEM TC-242-MDC 2015).

Figure 1 shows the plots of computed shrinkage strains using the final shrinkage model versus the measured shrinkage strains from the database. Similar plots have been generated for all compared models, but are excluded here for brevity; please refer to the thesis to be published soon by Timothy Clement, University of Minnesota Duluth.

Table 1. Coefficients of Variation for Various Shrinkage Models

Model	**Proposed**	**ACI 209**	**GL2000**	***fib* 2010**	**B4s**
Overall	0.377	0.465	0.391	0.478	0.439
Autogenous	0.610	N/A	N/A	0.907	0.791
Drying	0.480	N/A	N/A	0.617	0.492
Total	0.362	0.465	0.391	0.441	0.424
Swelling	0.723	N/A	N/A	0.948	0.863

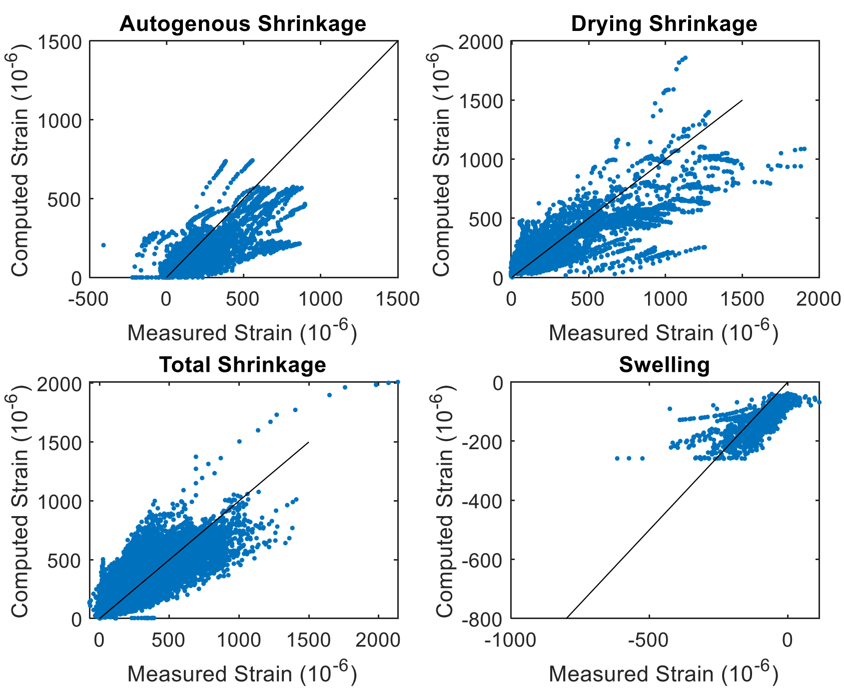

Figure 1. Final shrinkage model, computed strains versus measured strains

Table 2 summarizes the coefficients of variation for creep compliance computed using the final creep model and each of the historical models. The startling discrepancy in the values for B4s are because the dataset used to compute these coefficients of variation included some tests with temperature ranges far outside the applicable range for B4s (for example, tests conducted at 70°C). Ironically, the models with no temperature correction (ACI 209 and GL2000) performed better than B4s for these tests, as the B4s temperature correction procedure appeared to dramatically overcompensate for temperatures above 40°C. If these high temperature tests were removed from consideration, the B4s coefficients of variation more closely matched those from comparable models, but were still not lower than those of the proposed model. Overall, the proposed creep model consistently has lower coefficients of variations than all comparable historical models. Again, the superiority of the proposed model even holds when computing the coefficient of variation for the more limited dataset that conforms to the published limits of applicability of the B4 model (RILEM TC-242-MDC 2015).

Figure 2 shows the plots of computed compliance using the final creep model versus the measured compliance from the database. Similar plots have been generated for all compared models, but are excluded here for brevity; please refer to the thesis to be published soon by Timothy Clement, University of Minnesota Duluth.

Table 2. Coefficients of Variation for Various Creep Models

Model	Proposed	ACI 209	GL2000	fib 2010	B4s
Overall	0.313	0.443	0.391	0.444	7.26*
Total	0.286	0.411	0.355	0.410	1.33*
Basic	0.354	0.485	0.443	0.485	12.03*

* High CoV due to presence of high temperature tests (> 40°C) that temperature correction procedure of B4s cannot properly handle

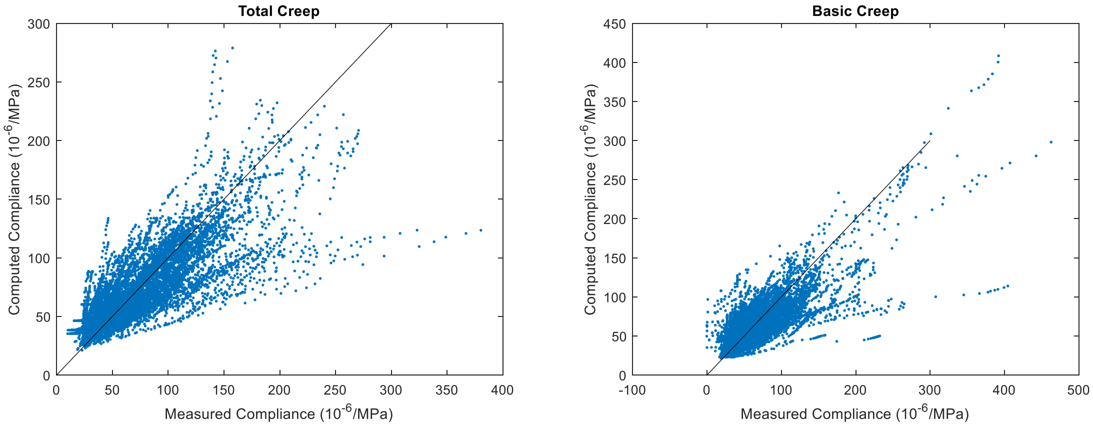

Figure 2. Final creep model, computed compliance versus measured compliance

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